Abstract

Let X be a metric-measure space equipped with Euclidean metric and a measure μ. We consider a non-negative and self-adjoint operator L on $L^2(X,\mu)$. Let T_t denotes the semigroup $\exp(-tL)$ related to L. The Hardy space is defined as follows. We say that a function $f \in L^1(X,\mu)$ belongs to the Hardy space $H^1_L(X,\mu)$ if and only if the maximal function $\sup_{t>0} |T_t f|$ belongs to $L^1(X,\mu)$. In the thesis we study three main topics concerning the Hardy space $H^1_L(X,\mu)$.

In the first part we obtain atomic decomposition theorems for $H^1_L(X,\mu)$. This means that each $f \in H^1_L(X,\mu)$ can be represented as series $f = \sum_k \lambda_k a_k$, where $\lambda_k \in \mathbb{C}$, $\sum_k |\lambda_k| < \infty$ and a_k are atoms. The atoms satisfy certain simple localization, size, and cancellation conditions. In the context studied here the atoms shall be related to a family of cubes $\mathcal{Q}$ that is a covering of X and will be defined as follows. A function a is called an atom, if there exists a cube $K \subset Q \in \mathcal{Q}$ such that: supp $a \subset K$, $\|a\|_\infty \leq \mu(K)^{-1}$, $\int a \, d\mu = 0$ or, alternatively, $a = \mu(Q)^{-1} \mathbb{1}_Q$ for $Q \in \mathcal{Q}$. For certain operators L we find proper coverings $\mathcal{Q}$ such that $H^1_L(X,\mu)$ admits atomic decompositions with atoms as above. We are particularly interested in the atomic decompositions of the Hardy space related to multidimensional versions of the classical Bessel, Laguerre, and Schrödinger operators.

Second question concerns characterizing the Hardy space $H^1_L(X,\mu)$ by certain singular operators. In our background we shall use the Riesz transforms that are formally defined as $R_j = (\frac{\partial}{\partial x_j} + V_j(x_j))L^{-1/2}$, $j = 1, \ldots, d$. With certain assumptions on L we prove that $f \in H^1_L(X,\mu)$ if and only if $f, R_1 f, ..., R_d f \in L^1(X,\mu)$.

The third question is related to multiplier theorems for the multidimensional Bessel operator B. We consider the spectral multipliers $m(B)$ and assume that a function m satisfies the classical Hörmander condition. We prove that the operator $m(B)$ is bounded from $L^1(X,\mu)$ to $L^{1,\infty}(X,\mu)$ and from $H^1_B(X,\mu)$ to $H^1_B(X,\mu)$. Additionally, we analyze the imaginary powers of B^{ib}, $b \in \mathbb{R}$ and show that our multiplier theorem is sharp.