Abstract

We study canonical quotients in model theory, mainly stable quotients of type-definable groups and invariant types in NIP theories.

The main results of the thesis are the following:

- We solve two problems from [HP18] concerning maximal stable quotients of groups type-definable in NIP theories. The first result says that if G is a type-definable group in a distal theory, then $G^{st} = G^{00}$ (where G^{st} is the smallest type-definable subgroup with G/G^{st} stable, and G^{00} is the smallest type-definable subgroup of bounded index). In order to get it, we prove that distality is preserved under passing from a theory T to the hyperimaginary expansion T^{heq}. The second result is an example of a group G definable in a non-distal, NIP theory for which $G = G^{00}$ but G^{st} is not an intersection of definable groups. Our example is a saturated extension of $(\mathbb{R}, +, [0, 1])$. Moreover, we make some observations on the question whether there is such an example which is a group of finite exponent. We also take the opportunity and give several characterizations of stability of hyperdefinable sets, involving continuous logic.

- For a NIP theory T, a sufficiently saturated model $\mathfrak{C}$ of T (so-called monster model), and an invariant (over some small subset of $\mathfrak{C}$) global type p, we prove that there exists a finest relatively type-definable over a small set of parameters from $\mathfrak{C}$ equivalence relation on the set of realizations of p which has stable quotient. This is a counterpart for equivalence relations of the main result of [HP18] on the existence of maximal stable quotients of type-definable groups in NIP theories. Our proof adapts the ideas of the proof of that result, working with relatively type-definable subsets of the group of automorphisms of the monster model as defined in [HKP21].

- We define the continuous modelling property for first-order structures and show that a first-order structure has the continuous modelling property if and only if its age has the embedding Ramsey property. We use generalized indiscernible sequences in continuous logic to study and characterize n-dependence for continuous theories and first-order hyperdefinable sets in terms of the collapse of indiscernible sequences.

- We study maximal WAP and tame (in the sense of topological dynamics) quotients of $S_X(\mathfrak{C})$, where $\mathfrak{C}$ is a monster model of a complete theory T and X is an $\emptyset$-type-definable set. Namely, let $F_{\mathrm{WAP}} \subset S_X(\mathfrak{C}) \times S_X(\mathfrak{C})$ be the finest closed $\mathrm{Aut}(\mathfrak{C})$-invariant equivalence relation on $S_X(\mathfrak{C})$ such that the flow $(\mathrm{Aut}(\mathfrak{C}), S_X(\mathfrak{C})/F_{\mathrm{WAP}})$ is WAP, and let $F_{\mathrm{Tame}} \subset S_X(\mathfrak{C}) \times S_X(\mathfrak{C})$ be the finest closed $\mathrm{Aut}(\mathfrak{C})$-invariant equivalence relation on $S_X(\mathfrak{C})$ such that the flow $(\mathrm{Aut}(\mathfrak{C}), S_X(\mathfrak{C})/F_{\mathrm{Tame}})$ is tame. We show good behaviour of F_{WAP} and F_{Tame} under changing the monster model $\mathfrak{C}$. Namely, we prove that if $\mathfrak{C}' \succ \mathfrak{C}$ is a bigger monster model, F'_{WAP} and F'_{Tame} are the counterparts for F_{WAP} and F_{Tame} computed for $\mathfrak{C}'$, and $r : S_X(\mathfrak{C}') \to S_X(\mathfrak{C})$ is the restriction map, then $r[F'_{\mathrm{WAP}}] = F_{\mathrm{WAP}}$ and $r[F'_{\mathrm{Tame}}] = F_{\mathrm{Tame}}$ Using these results, we show that the Ellis groups of $(\mathrm{Aut}(\mathfrak{C}), S_X(\mathfrak{C})/F_{\mathrm{WAP}})$ and $(\mathrm{Aut}(\mathfrak{C}), S_X(\mathfrak{C})/F_{\mathrm{Tame}})$ do not depend on the choice of the monster model $\mathfrak{C}$.

The results contained in the first, second and fourth bullets are joint with Krzysztof Krupiński and the ones contained in the third bullet are mine alone. The results in the

first bulled come from [KP22], in the second from [KP23], the results in the third bullet will be contained in a future paper by myself, and the results in the last bullet will be contained in a future joint paper with Krzysztof Krupiński.

References

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