Stochastic analysis of Lévy systems

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Abstract

The Lévy system formula is a way to describe accumulated effect of jumps of a Markov process by means of the corresponding measure of intensity of jumps. Lévy systems proved to be an important tool in potential theory of Lévy processes and in other applications, including L^p estimates of Fourier multipliers. The dissertation focuses on Lévy systems of Lévy processes. Our initial motivation was to better understand and advance the stochastic analysis to study martingales, semigroups and Fourier multipliers related to the processes.

The main focus of Section 1 of the dissertation is on multiple or iterated versions of Lévy systems. We hope that our straightforward approach is at least of didactic interest and also gives motivation for later developments. In fact, the identities for iterated *mixed* Lévy systems prove to be rather involved, and Section 1 only provides them in the two-fold variant.

In Section 2 we use a martingale variant of Lévy systems to study Fourier multipliers defined by means of Lévy processes. Fourier multipliers are an important topic in harmonic analysis, with contributions from stochastic analysis and deep relationships to martingale theory. It is of particular interest to estimate the operator norms of the multipliers on L^p spaces given their symbols. We use the stochastic calculus presented in Section 1 as one of the tools. Section 2 appeared in print.

It is well-known that the jumps of a Lévy process may be described in terms of Poisson random measures with intensity or control measure given by the Lévy measure of the process. In Section 3 we show that Lévy systems for Lévy processes may be considered as special cases of *iterated* Mecke-Palm formulas, which we derive there for general Poisson random measures. We thus complement the results of Section 1. The involved combinatorics of *configurations* in Section 3 sheds light on our difficulties in Section 1. The Mecke-Palm formulas also give moment formulas for (iterated) stochastic integrals against Poisson random measures, which extend recent results.