Report for the dissertation by Konrad Krystecki ‘Ruin probability in multidimensional self-similar Gaussian risk models’

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The dissertation focuses on the asymptotic properties of multidimensional Gaussian processes, specifically in the context of stochastic risk models. Gaussian processes are central to many applications due to their mathematical tractability and relevance across diverse disciplines. The main contributions of the thesis are about the asymptotic behavior of ruin probabilities. The analysis is motivated by theoretical questions in extreme value theory and practical questions in fields such as finance, insurance, and queuing systems.

There is a vast literature on ruin probabilities in stochastic systems. In Gaussian setting, the majority of it is devoted to the models driven by one-dimensional Brownian motion. This dissertation extends the scope to more complex settings, like fractional Brownian motion and multidimensional models.

Chapter 1 gives an introduction to the dissertation and briefly summarizes the main findings. Chapter 2 explores simultaneous ruin probabilities in two-dimensional Brownian motion with correlated components. Both drift and barriers are of the form $* \, u^c$ and the asymptotic behavior as u inf is studied. It appears that there are several essentially different regimes, determined by the relationships between the parameters, and the thesis provides a comprehensive analysis of each of them.

Chapter 3 focuses on the analysis of risk processes driven by fractional Brownian motion, deriving the asymptotics of ruin probabilities under random inspection times. These findings are particularly significant, as the literature on ruin probabilities in non-Markovian settings remains relatively sparse. Moreover, they have considerable practical value, especially in applications to queueing systems.

Chapter 4 studies Parisian-type ruin probabilities for two-dimensional Brownian motion $\left(W_1^*(t), W_2^*(t)\right)$ with linear drift. Specifically, the asymptotic behavior of probability

$$\mathbb{P}\left(\exists s', t' : \min_{s\in[s', s'+H_1(u)]} W_1^*(s) > u, \min_{t\in[t', t'+H_2(u)]} W_2^*(t) > au\right).$$

is investigated. It appears (perhaps, unsurprisingly) that the asymptotic behavior is different in the case where $H_1(u)$ and $H_2(u)$ vanish as $u \to \infty$ and in the case where they do not vanish. Both cases are studied in detail in the dissertation. The chapter is supplemented with simulations illustrating the findings.

Chapter 5 addresses multidimensional models based on positively correlated Brownian motions with linear drift. It derives asymptotics for non-simultaneous ruin probabilities, which coincides with the two-dimensional results found earlier.

Overall, the dissertation presents results that are both accurate and rigorously proven. It is well-structured and self-contained. The inclusion of remarks and examples enhances the clarity of the findings. Finally, simulations conducted as part of this work validate the theoretical findings, illustrating their practical relevance. The dissertation is written in clear and fair English, which, combined with its thoughtful organization, makes it accessible and enjoyable to read.

The reference list is extensive and appropriately curated. While it is impossible to reference all related studies, the author finds a balance by focusing on those most significant and closely aligned with the dissertation's contributions. This demonstrates a strong ability to engage with existing knowledge, effectively integrating it and producing new, meaningful results.

Several remarks can be made, though none are critical and do not diminish the overall positive impression of the dissertation. The most significant remarks are presented first.

- There is a gap in the proof of Proposition 3.2.1. The last displayed formula should involve the conditional density of X_k given $\tau = k$ rather then f_{X_k} (the following corollary does speak about the conditional distribution while writing the same incorrect formula). The former can be unbounded in k, and it is in fact in the Poisson (exponential) case it is equal to $\frac{k}{T}$.
- The statement of Proposition 3.2.1 is flawed in several ways. Firstly, the distribution of X_τ does not have density, since it obviously has an atom at zero corresponding to the event $X_1 > T$. So the proposition should in fact address the continuous component of the distribution. Secondly, τ depends on T, so the writing '$f_{X_\tau} \in (0, \infty)$ for any $T \in (0, \infty)$' is a bit misleading: it looks as it were about single distribution, which is not the case.
- The computation in Corollary 3.2.3 is incorrect. The correct resulting density is $\lambda e^{-\lambda T}$, which is actually quite easy to see from the first principles.
- In many asymptotic results, the complementary standard normal cdf Ψ is used. However, for the practical purposes, it would be better to use the asymptotic $\Psi(x) \sim \frac{\varphi(x)}{x}$ as $x \to \infty$.
- Throughout the dissertation, there is a colon following the supremum sign, which is quite odd.
- Formula (1.1): what is $\mathcal{S}_t$?
- Chapter 2 uses $\sim$ sign in many formulas containing several parameters, which results in certain ambiguity. It would not harm to admit at the beginning that in all cases this refers to the asymptotic behavior as $u \to \infty$.
- Page 12: the last line in the first displayed equation comes out of the blue, so it is not clear whether it is correct or not. If it is correct, then there should be $\frac{\boldsymbol{c}}{\sqrt{2}}$ rather than $\boldsymbol{c}$ in the following line.

- Page 31: The statement that $\kappa > 0$ iff $t_0 > T$ is far from being straightforward (frankly, I do not see why it is true at all).
- Page 37: The important condition that Z_i are independent of B_H is missing. Further, the correct formula for τ_i seems to be $\tau_i = \sup\{i : X_i < X_{\tau_{i-1}}\}$. So $\tau_i = \tau_{i-1} - 1$, right? The same remark goes for the proof of Theorem 3.2.1, Case $H < \frac{1}{2}$.
- Page 42: It would be convenient if the statement of [35][Prop 3.1] were given in the dissertation.
- Page 49, bottom: 'two-dimensional time to be spent' sounds very confusing as well as the reference to $(H_1(u), H_2(u))$ as a barrier.
- Page 51, line 3: a comma is missing after 'functional', which makes the proposition difficult to read.

Conclusion

In view of the said, I believe that the dissertation is an important contribution, which advances the understanding of Gaussian risk models by addressing fundamental questions in the asymptotics of ruin probabilities. In my opinion, it meets all the requirements for a PhD thesis and I recommend to the PhD committee that the PhD degree is awarded to Konrad Krystecki.

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