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Report on the thesis

"Asymptotic properties of models governed

by the smoothing transform"

submitted by Piotr Dyszewski

The thesis is based on four papers. They treat different models but they have a common theme. The common theme is the the tail of the distribution which solves a certain stochastic recursion equation, often called smoothing transform.

The first paper, written in collaboration with Ewa Damek, considers iterated random functions, the main example being the so-called random difference equation. The solution of the recursion satisfies the so-called stochastic fixed point equation. and can be seen as the stationary distribution of a Markov chain. The study of such solutions is a classical topic, going back to Harry Kesten and Charles Goldie, which still attracts a lot of recent attention. The paper gives conditions for the existence of a solution and characterizes the decay of the tail of the solution. More precisely, the tail is compared with the the tail of an affine approximation to the recursion.

The second paper, written in collaboration with Gerold Alsmeyer, considers the fixed point of a non-homogeneous smoothing transform. There are several models leading to such fixed points, for instance one can study perpetuities, the lengths of minimal spanning trees or weighted branching processes, to name a few. The most prominent example is the number of steps needed by the "Quicksort" algorithm. The recursive nature of this algorith allows to write a stochastic recursion for the number of steps, this obersvation goes back, I think to Uwe Rösler. Under suitable conditions, it is known that there is a unique solution (within the class of centered distributions). The authors provide conditions such that the solutions have (some) finite exponential moments. Moreover, they are able to say in particular cases, which exponential moments exist and which don't.

The third paper, written in collaboration with Dariusz Buraczewski, investigates branching processes in random environment. This is a variant of the classical Galton-Watson process where the law of the offspring varies over time, more precisely these offspring laws are chosen according to an i.i.d. sequence called "environment". The authors give a precise asymptotics for the probability of an upper large deviation, complementing a series of papers giving only the exponential decay rates of such probabilities. In the subcritical case, the authors study the first passage time, i.e. the first time when the population exceeds a certain size. They give precise asymptotics for the probabilities of upper and lower deviations for the first passage time. Finally, in the subcritical case, the total population size is a finite random variable , which can be approximated by the total population sizes up to time n. The authors prove precise asymptotics for the upper deviation probabilities of these population sizes and also for the corresponding first passage times. This is an impressive work unifying and extending previous results. The proofs combine a clever use of the recursive structures with large deviation arguments. Although the details are technical, the heuristic is explained very well. However, to go from the heuristic to the precise proofs invoves a considerable amount of hard work.

The fourth paper, written in collaboration with Dariusz Buraczewski, treats a one-dimensional random walk in an i.i.d. random environment. This model goes back to Fred Solomon (1975) and has been studied quite intensively in the last two decades. Recurrence and Transienca and the classification of ballistic and subballistic regimes go back already to Fred Solomon. Limit laws were proved in famous papers by Harry Kesten and by Yakov Sinai. In 1996, there was a new wave of interest, sparked by the paper "Tail estimates for one-dimensional random walk in random environment" by Amir Dembo, Yuval Peres and Ofer Zeitouni [DPZ], where a subexponential decay rate for slowdown probabilities in the ballistic regime was proved. The authors improve the result of [DPZ] and replace the logarithmic asymptotics of [DPZ] by a much more precise statement. They also give precise decay rates for the slowdown probabilities in the subballistic regime. The main idea of the proof is a representation of the hitting times of the random walk in random environment by a branching process in random environment with immigration. This representation goes back to Harry Kesten, M. V. Kozlov and Frank Spitzer. This reduction to the investigation of a branching process in random environment allows to apply the results (and also the methods) of the third paper. Again, parts of the proofs are technical and I would not claim to have followed all the details, but the main lines of the proofs are made clear, and the presentation is reader-friendly.

The thesis is technically on a high level and the author demonstrates a broad knowledge of the field. In an active area with considerable recent progress, the thesis gives new and interesting results. I was impressed by the variety of models fitting in the common theme.

The author knows how to put his results in a general context. He has clearly read a lot of the literature and he can already be considered an expert in the topic. The author masters the methods used in the field and he is able not only to apply but also to further develop them.

I full-heartedly recommend the acceptance of the thesis. The results but also the presentation go far beyond what one can expect from a PhD thesis, and I propose to grade the thesis with

“passed with high distinction” (”summa cum laude”).

With best regards,

Prof. Dr. Nina Gantert