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**Report on the PhD thesis *Bounded invariant equivalence relations*
by Tomasz Rzepecki**

The subject matter of the thesis is, by now, a well established theme connecting the study of strong types in Model Theory with Descriptive Set Theory and Topological Dynamics. The first work relating strong types to descriptive set theory was [CLPZ01]. The area deepened with a result of Newelski [New03]. It was developed further and more explicitly tied with the notion of Borel cardinality in [KPS13]. (Borel cardinality has been a central notion in Descriptive Set Theory in the last quarter of a century.) An important result certifying that the connection with Borel cardinalities is real and deep was proved by Kaplan, Miller and Simon [KMS14]. They showed that smoothness on the Descriptive Set Theory side corresponds precisely to type definability on the strong types side. Further work on this connection and the discovery of an important role of Topological Dynamics was made by Krupiński, Pillay, and the author of the thesis [KPR15, KP17b, KR16, KR18]. The work included in the PhD thesis, especially the results presented in Chapter 5, brings this string of developments to a satisfactory conclusion. In the thesis, an abstract topological/dynamical/descriptive set theoretic framework is found that allows the author to prove a general abstract theorem (Theorem 5.52) of which the model theoretic results are particular instances.

As should be clear from the previous paragraph, the thesis is current and topical. Furthermore, the area of research described above is mathematically important and well worth developing. The abstract treatment given by Mr. Rzepecki is a significant contribution to this area. It is surprising that such an abstract approach succeeds and gives sharp consequences. Some notions, especially those introduced in Chapter 7, like orbitality or weak orbitality, may have broader applicability as they seem interesting, for example, from the point of view of Descriptive Set Theory (with the obvious change in this context of compact to Polish). These notions have not been investigated so far in the setting of Polish group actions and, for example, their impact on the complexity of the induced orbit equivalence relations may merit attention. This is less true about the core notions of the study from Chapter 5, even though the condition of uniformity in
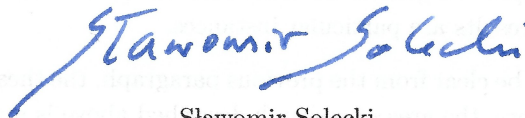
the definition of uniformly properly group-like is certainly intriguing. Further, replacing the ad hoc argument of Kaplan/Miller/Simon on Borel reducing $\mathbb{E}_0$ by representing the equivalence relation under study as the image of the coset equivalence relation of a compact group by an F_σ subgroup (as in Theorem 5.52) is mathematically desirable, especially if it is accomplished in an abstract framework.

To point out some shortcomings, as there always are some, one has to remark that the abstract treatment from Chapter 5 is not immediately mathematically appealing from the point of view of Topological Dynamics or Descriptive Set Theory. (But see the remarks above about certain notions introduced in Chapter 7.) So, quite possibly, the treatment in its current form will not have an independent life outside of applications to Model Theory, at least for now. Also, the fundamental problem of finding a characterization of Borel cardinalities attained by the σ-compact equivalence relations associated with strong types remains open, and the new point of view has not yet contributed to shedding light on it.

The thesis is well composed (the progression of arguments is well motivated and illustrated with examples) and carefully written (I found only a handful of misprints). The inclusion of Chapter 2 containing known, but not easily accessible, results was much appreciated by this referee. In fact, because of the breath of the presentation, making the thesis more widely available would be a service to the broader mathematical community.

Overall Mr. Rzepecki's thesis fulfills all the requirements placed on a PhD dissertation. I am happy to recommend awarding the PhD degree to Mr. Rzepecki.

Sincerely yours,

Sławomir Solecki