A simple approach to entire normalized solutions to an elliptic Schrödinger equation in the L^2-(super)critical and Sobolev-critical case

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Schrödinger-type equations model a lot of natural phenomena and their solutions have interesting and important properties. This gives rise to the search for *normalized solutions*, i.e., when the $L^2(\mathbb{R}^N)$ norm is prescribed. We propose a simple and novel approach, based on a minimization argument and introduced in [1], to study problems of the form

$$\begin{cases} -\Delta u + \lambda u = g(u), \\ u \in H^1(\mathbb{R}^N), \\ \int_{\mathbb{R}^N} u^2 \, \mathrm{d}x = \rho^2, \end{cases}$$

where $N \geq 3$, $\rho > 0$ is given a priori, $\lambda \in \mathbb{R}$ is part of the unknown, and $g\colon \mathbb{R} \to \mathbb{R}$ has Sobolev-critical growth at infinity and L^2-(super)critical and Sobolev-subcritical growth at the origin.

This talk is based on joint work with Jarosław Mederski [2].

References

[1] B. Bieganowski, J. Mederski: *Normalized ground states of the nonlinear Schrödinger equation with at least mass critical growth*, J. Funct. Anal. **280** (2021), no. 11, 108989.

[2] J. Mederski, J. Schino: *Least energy solutions to a cooperative system of Schrödinger equations with prescribed L^2-bounds: at least L^2-critical growth*, arXiv:2101.02611v2.