On the empirical spectral distribution for the certain generalizations of the sample covariance matrices

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Given $N, m \in \mathbb{N}$, we consider random matrices of the form

$$\mathcal{L}_{N,m} = \sum_{\alpha=1}^{m} \xi_\alpha Y_\alpha Y_\alpha^T \quad \text{and} \quad \mathcal{A}_{N,m} = \sum_{\alpha=1}^{m} \xi_\alpha Y_\alpha X_\alpha^T,$$

where ξ_α, Y_α, X_α, $\alpha \in [m]$, are mutually independent, $\mathbf{E}\xi_\alpha = \mu_N \geq 0$, $\mathbf{Var}\xi_\alpha = o(1)$ as $N \to \infty$, and $Y_\alpha, X_\alpha \in \mathbb{R}^N$ are random vectors with covariance matrices depending on $\alpha \in [m]$. We study the asymptotic behaviour of empirical spectral distributions of these matrices in two following regimes as $N \to \infty$: (1) $\mu_N \to \mu > 0$, $m/N \to c > 0$ and (2) $\mu_N \to 0$, $\mu_N m/N \to c > 0$. In particular, for a certain choice of covariance matrices of vectors Y_α, X_α, $\alpha \in [m]$, we prove the weak convergence of the empirical spectral distributions of $\mathcal{L}_{N,m}$ and $\mathcal{A}_{N,m}$ to some non-random probability measures related to the Marchenko -Pastur law and to the Wigner semicircle law. The talk is based on the joint work with Alicja Dembczak-Kołodziejczyk.